where $\phi(r)$ is the maximum value of the function on the circle in question. When ρ is integral, this approximation becomes

$$\frac{\log \phi(r)}{\log r}.$$

A number of theorems are given relating to the order of functions combined from functions of given order.

It is shown that a function is of the same order as its derivative, and that from the knowledge of the zeros of a function we may derive an approximation to the large zeros of its derivative.

If a function is of "genre" p, its derivative can at most have only p zeros beyond those given by the extension of Rolle's theorem.

The memoir closes by suggesting other applications of the theory.

"On Areal Induction." By GEORGE J. BURCH, M.A., F.R.S., Reading College, Reading. Received September 10, 1901.

In my paper "On the Relation of Artificial Colour-blindness to Successive Contrast,"* I described a modification of some experiments by Sigmund Exner.

A disc, half black and half white, with a very narrow slit at the junction of the two halves, revolves in front of an incandescent lamp. Another lamp throws a light upon the face of the disc. While it is rotating slowly, the incandescent filament of the lamp, seen through the slit as it passes across, looks bright against the white card, but when a certain velocity is reached it appears as a black thread against a brighter background.

I suggested as the explanation of this phenomenon that the intense light of the filament induced an amount of fatigue out of all proportion to the sensation excited by it, so that the less fatiguing illumination of the white card produced a greater effect on the senses than the sum of the sensations due to the filament and the subsequent light.

In a recent paper,† however, Dr. Shelford Bidwell has described an experiment having an important bearing on the subject.

By a suitable arrangement a disc of green light is projected on a screen; this is followed by a somewhat larger disc of white light, and the cycle is completed by an interval of darkness. The novelty of the experiment consists in the addition of a black spot in the centre of the white disc, so that when the apparatus is worked the site of this spot is illuminated five or six times a second by green light only,

* 'Roy. Soc. Proc.,' vol. 66, p. 212.

† 'Roy. Soc. Proc.,' vol. 68, p. 277.

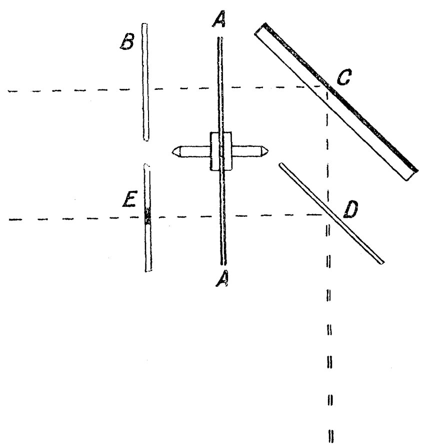
whereas on the surrounding portions each green flash is followed by a white one. The area corresponding to the green disc appears purple, with a black spot in the middle.

Obviously this experiment falls under the first of the four classes of successive contrast described in my paper,* *i.e.*, the first stimulus excites a single colour-sensation, and the second excites all the sensations. The conditions are less simple than those of the second, third, or fourth classes described by me, and I have therefore repeated Dr. Shelford Bidwell's experiment, and have extended the same principle to the other classes of contrast referred to.

The following arrangement serves admirably for exhibiting the phenomenon. In front of the disc A is placed a piece of coloured glass (preferably green or blue) B, and behind it, at an angle of 45° to its plane, the silvered glass mirror C. Light passing through B is reflected from C parallel to the plane of the disc. In the path of the reflected rays is a second mirror, D, of unsilvered glass, so placed as to reflect rays from E in the same direction. E may conveniently be a black bead fixed on a wire or a black spot on a piece of glass.

The disc A, fig. 1, has two opposite sectors of about 30° cut out, and the positions of B and E are so regulated that on rotating the disc a flash of coloured light from B is seen first in the mirror, and then a

Fig. 1.



flash of white light of less intensity is reflected from D, and in the midst of it the image of the black bead, this being followed by a longer period of darkness. As long as the rate is not much more than six revolutions per second the light, when green glass is used, appears

* 'Roy. Soc. Proc.,' vol. 66, p. 267.

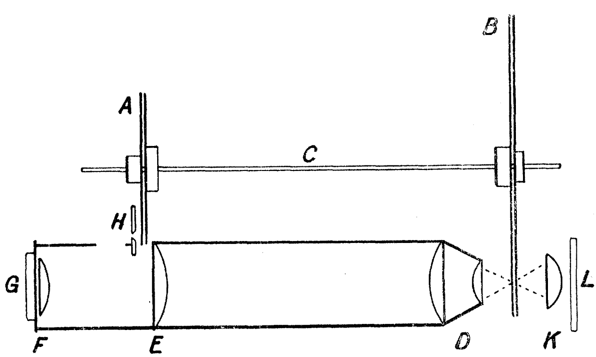
purple, and the bead looks quite black against it. But when the disc is driven faster, at a certain speed both bead and glass look green, turning black and purple again respectively when the speed diminishes. If the intensity of the flash is reduced, there comes a point at which the phenomenon ceases to be observed, and if the area of the spot is increased, a limit is reached beyond which the effect no longer extends to its centre. My results agree as to these details completely with those obtained by Dr. Shelford Bidwell.

The essential difference between this experiment and those described in my paper consists in the use of the black spot, and the fact discovered by Dr. Shelford Bidwell, that the contrast effect may be so powerful in a spot not directly subjected to the first stimulus, is of the utmost importance, proving, as he points out, that the first sensation is not merely swamped by the second, but actually never comes into existence.

I do not, however, agree with him in thinking that no explanation of it can be afforded by the Young-Helmholtz theory, nor do I consider it warrants us in postulating an independent white sensation, and this view is supported by the following experiments, made on the lines laid down in my previous paper.*

Two discs A and B, fig. 2, are carried by a shaft C about 100 cm. long. Each disc is composed of two equal circular sheets of metal, with a pair of sectors of 30° cut out at opposite ends of a diameter, so

FIG. 2.



that by rotating one over the other the aperture may be of any desired width, and fitted with clamping nuts so that the relative position of A and B on the shaft can be varied. D is an objective of 7 cm. focal length, and E is a plano-convex lens 10 cm. in diameter, which with the smaller lens F forms in effect a Campani eye-piece of about

* 'Roy. Soc. Proc.,' vol. 66, p. 209.

12·5 cm. focal length. G is a Thorp's replica of Rowland's gratings, 15,000 lines to the inch, giving a spectrum of the slit H apparently in the centre of the field of view of the optical system formed by the lenses D, E, F, which constitutes a kind of microscope, with the lens K serving as condenser. Behind K is placed a screen L of suitable coloured glass, or a train of prisms for illuminating the apparatus with spectral light.

On rotating the shaft the disc A allows a narrow spectrum of about 10° dispersion and 1° wide to appear momentarily, and immediately afterwards a bright flash of monochromatic light fills the field.

As in my paper on Simultaneous Contrast,* a strip of black material is pasted on the plane side of the lens E to serve as a background for the spectrum, which should exactly cover it. The part, therefore, of the retina on which the image of the spectrum falls is not directly stimulated by the second flash, which fills the rest of the field. The results show, however, that it is indirectly affected as in Dr. Shelford Bidwell's experiments.

With a screen of ruby glass the spectrum corresponds with that seen during temporary red blindness, *i.e.*, the red is missing, and the orange and yellow are replaced by green. With a blue screen composed of a film stained with Prussian blue, and a pale cobalt, or, better, a gelatine film stained with aniline blue, the blue and violet disappear, and the green is weakened, but the red is bright. The addition to these of a yellow glass causes the violet of the spectrum to reappear. And the same effect is produced by a green glass. In a word, the results are exactly similar to those described in Section II, paragraph 3, of my paper on Successive Contrast, the difference between the two experiments being that in the new one the retinal area on which the contrast effect, or colour-blindness, is produced, not being exposed to the direct action of the second stimulus, it becomes possible to use a series of rapid flashes instead of a single one, and the effect appears practically continuous.

But the difference between this and Dr. Shelford Bidwell's experiment consists in the use by him of a second stimulus compounded of all the colours, and by me of a second stimulus restricted to one colour. The first step in the discussion must be to ascertain the relation between the results obtained when the second stimulus is white, and when the second stimulus is of the same colour as the first.

In every experiment I have made, whether with one double flash or with a series of such flashes, the result has been the production of a considerable degree of momentary blindness to the colour or colours composing the second flash of the cycle. And this statement is equally true if the two flashes are of different colours. Whatever colour-sensations are common to the two flashes are wholly or partly blotted

* 'Roy. Soc. Proc.,' vol. 67, p. 227.

out, and the remainder are combined in the resulting sensation. But inasmuch as the optimum period of the cycle is different for each colour-sensation, the use of pigment colours leads sometimes to apparently divergent results, which, however, are easily accounted for when the spectra of the pigments are taken into account.

In manipulating the apparatus when both flashes are monochromatic a farther clue to the explanation is obtained. To produce the maximum Exner-effect, the second stimulus must be much less intense than the first, though it may last longer. This accounts for the difference of the appearance when the direction of the rotation is reversed, since, unless the relative intensity and duration as well as the colours of the flashes are interchanged, the conditions under which the phenomenon occurs are no longer observed.

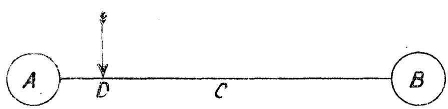
I fail to see why an independent white sensation must be postulated. If the first stimulus is green and the second white, then, on Young's hypothesis, the second stimulus consists of red, green, violet (and blue according to my own experiments). Accordingly, the green, which is common to both stimuli, vanishes, and the eye perceives that mixture of red, blue, and violet so familiar in purple pigments and flowers. It appears to me that Dr. Shelford Bidwell's black-spot experiment adds farther confirmation to this explanation, since it shows conclusively that the response to the first stimulus is inhibited—if that word may be borrowed—by a second stimulus falling upon a neighbouring part of the retina. For the second stimulus, white, must necessarily include the same physical stimulus, green, as the first, and this component of it is by itself, as my experiments show, quite competent to produce the observed result, namely, the inhibition of the response to the first stimulus. Moreover, a second stimulus which contains all the elements of white save green does not inhibit the response to green, and the same is true of the other colours. If the first stimulus is red and the second blue, or if the first is green and the second purple, or *vice versâ*, *i.e.*, if neither stimulus includes any of the components of the other, the effect is nil. It is difficult to see why this should be the case if the phenomenon is a function of a white sensation. To me it seems more simple to explain the whole phenomenon as follows :—

It is granted that a diminution of the intensity of a sensation occurs whenever the stimulus is prolonged. This diminution may occur so suddenly under suitable conditions that it becomes difficult to regard it as the mere result of using up material previously stored. It resembles rather those reflexes by which the organs of vision are protected from a sudden light, namely, the winking of the eyelids and the contraction of the iris, and is probably a provision for preventing unnecessary waste of material.

The question then arises whether such a function would be located

in the central or in the peripheral organ. Let A, fig. 3, represent a peripheral organ, B a central organ, and C the nerve connecting them. Let a stimulus, E, applied at A evoke a sensation, S, at B. Similarly, let another stimulus, E_2, applied at A evoke a corresponding sensation, S_2, at B. Then the experimental facts may be stated thus: if there is a certain time-interval of $\frac{1}{70}$ of a second between E_1 and E_2, and a certain relation between them of intensity and duration, the sum of the resulting sensations S_1 and S_2 may be practically zero, or, at any rate, very much smaller than either sensation by itself.

FIG. 3.



(1.) The first obvious deduction is that made by Exner, viz., that a period of at least $\frac{1}{70}''$ must elapse between the application of the stimulus at A and the arousing of the sensation at B.

(2.) It is also evident that this delay of $\frac{1}{70}''$ is not simply the transmission time of the nerve-response along C from A to B, for, if it were, the second impulse would reach B too late to affect the first, unless we suppose either a double track along C or the propagation along a single track of a double response, with components travelling at different rates—a hypothesis not supported by any facts within my knowledge.

(3.) If the delay of $\frac{1}{70}''$ occcurs at B we must also suppose either a double track along C, or else that each nerve-fibre is capable of transmitting two opposite kinds of response, namely, one with a latency of $\frac{1}{70}''$ arousing sensation, and the other deadening it, and acting without delay.

(4.) It seems simpler to suppose that this delay of $\frac{1}{70}''$ occurs somewhere on the track between A and B, and probably close to A, *e.g.*, at D, and that it is quite distinct from any delay due to transmission.

(5.) Not only do these experiments prove that there is a delay, but that, under certain circumstances, a second stimulus may modify to the point of annulling it the response to a previous stimulus. It has been shown* that two stimuli in rapid succession may give rise in nerve to what appears to be a single response, but in that case the response is at least as strong as either stimulus would produce alone, whereas here the response is less than it would have been without the second stimulus.

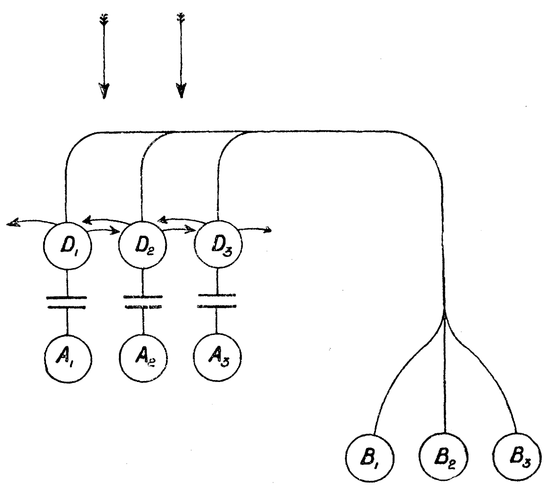
These facts point to the existence, probably in the retina, of some mechanism sensitive to light, the function of which is to regulate for

* Gotch and Burch, 'Journal of Physiology,' vol. 24, p. 410.

each component of the visual field the intensity of the visual impulse transmitted to the central organ, much as the iris and the eyelids regulate it for the whole. It is acknowledged that we cannot as yet attribute definite functions to the several elements of the retina, and that we cannot demonstrate a structural continuity between the three layers of which it is composed, although we may conclude that changes set going in the rods and cones sweep through the inner nuclear layer and issue along the fibres of the optic nerve as nervous impulses.* There is, therefore, room between the extreme peripheral structures and the beginnings of the optic nerve, for a structure, which when stimulated by light falling on it may block wholly or in part the impulses coming to it from the periphery.

In fig. 4 three similar hypothetical elements of the retina are drawn in diagram side by side. In each A represents the sensitive peripheral structure giving rise, when stimulated by light, to a visual impulse, and

FIG. 4.



D represents the controlling structure, also sensitive to light, the function of which is to block the visual impulse if necessary. D_1 is represented as having cross-connections with D_2, and *vice versâ*. Suppose a flash of light, F_1, coming in the direction of the arrow to fall suddenly on all three retinal elements. It gives rise in A_1, A_2, and A_3 to a condition of excitation—lasting longer than the flash itself, and dying gradually away, arousing in the central organ the sensation of light during its earlier and of the "positive after-effect" during its later stages. But at the same time D_1, D_2, and D_3 are excited, and

* Foster, 'Text-book of Physiology,' Part IV, p. 1203.

they exercise a momentary control over the visual impulses from A_1, A_2, and A_3, transmitting them through C to B with an intensity considerably reduced at first in the case of a bright flash. Perhaps even the apparent delay of $\frac{1}{70}''$ may be due to this initial reduction.

Next suppose a second flash falling on A_1 D_1 and A_3 D_3. Owing to the cross connections from D_1 and D_3 to D_2, the activity of D_2 is aroused again, and the visual impulse from A_2 is blocked once more, so that little or none of it gets through to B_2.

If we suppose with Young that each colour-sensation results from the activity of an independent peripheral system, it is easy to explain the experimental phenomena. Let the second stimulus be white and the first green. D_1 and D_3, having been made to act strongly by the repeated excitation, will block all visual impulses of green from A_1 and A_3 by direct action, and from A_2 by virtue of their cross-connections with D_2. The result will be that the components of purple, *i.e.*, $(W—G) = \{(R+G+B+V)-G\}$ will reach B_1 and B_3, and nothing at all will reach B_2—exactly as in Dr. Shelford Bidwell's experiment.

It is evident from a multitude of experiments that we have to deal with two classes of contrast effects, namely, those which are induced gradually and pass off slowly, and others which are momentary and suggest the term spasmodic. The latter require for their production a sharp and sudden stimulus, and one such stimulus may even be followed by a multiple response, as in the recurrent images of Purkinje or Charpentier's bands. To myself these phenomena recall very forcibly the multiple electrical response of a nerve-muscle preparation, or of an electrical organ, to a single strong stimulus, and I suggest that the "recurrent image" is caused by intermittent blocking of the positive after-effect. The failure of Talbot's law for low speeds is easily explained on this hypothesis. The ratio of the magnitude of the response of the "D" structures to the magnitude of the stimulus is greater when the stimulus is repeated a certain number of times per second, *i.e.*, a larger percentage of the total visual impulse from A is blocked by D when there are fifty flashes per second than when there are only five, and consequently the mean brilliancy appears less until the limit is reached, beyond which Talbot's law holds.

The phenomenon is manifested in greatest intensity when the stimulus consists of two or three flashes close together followed by an interval of rest. Possibly the reason of this is partly that the activity of D may be maintained, and partly that A may recover from its fatigue; however that may be, it is essential in all experiments on the reversal of the image that the eye should be in darkness for one-half of the cycle.

Another phenomenon explicable on this hypothesis is the flickering of certain spectral colours. If a small weight is held out at arm's length, after a time the muscles begin to shake and quiver. And if a

highly magnified spectrum is gazed at steadily in the neighbourhood of D, or F, or G, the colours begin to change and flicker exactly as if some controlling mechanism were getting fatigued. But if either of the contending sensations is *thoroughly* exhausted, as in artificial colour-blindness, this flickering ceases, and is abolished until the retina recovers. I have called attention to the fact that the positive after-effect or dazzle-tint of artificial colour-blindness does not undergo those cyclic changes of colour which have been so long familiar in ordinary after-images.*

To sum up—

(1.) There appears to be strong evidence of the existence in the retina, between the structures in which the visual impulses originate and the beginnings of the optic nerve, of a set of structures by which the intensity of the visual impulses transmitted to the central organ is regulated.

(2.) That these structures are sensitive to light, and when excited block wholly or in part visual impulses coming from the periphery.

(3.) That these structures are cross-connected so that the excitation of one affects those in its neighbourhood, thus increasing the contrast between a bright object and its surroundings, and probably preventing the blurring of an image by diffused light from the retina.†

(4.) That they may by sudden stimuli be excited to a very powerful or even a multiple response.

(5.) That many phenomena of contrast and of areal induction may be accounted for on such a hypothesis.

(6.) That Dr. Shelford Bidwell's experiments are explicable without assuming a white sensation.

"Further Observations on Nova Persei. No. 4." By Sir NORMAN LOCKYER, K.C.B., F.R.S. Received October 10, 1901.

The last paper‡ contained an account of the observations of the Nova made at Kensington between March 25 and May 7. The observations are, in the present paper, brought up to September 30. Between June 18 and August 8 no work was done on the Nova, owing chiefly to the interference of buildings and twilight.

* 'Phil. Trans.,' B, vol. 191, p. 7.

† I find no evidence of any such cross-connection between one colour sensation and another.

‡ 'Roy. Soc. Proc.,' vol. 68, p. 399.